CONSTRUCTIONS OF EXACT SOLUTIONS OF THE EQUATIONS OF GAS DYNAMICS IN THE PRESENCE OF DISCONTINUITIES

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In the paper by Korobeinikov and Riazanov [1] in order to find exact solutions with discontinuities, the particular solution of one-dimensional gas dynamics found by Sedov [2] was used.

Taking into account that the shock wave propagates into the gas at rest with initial variable density $\rho = \rho_1(r)$ and constant pressure p_1, the authors showed that the special difficulty of the given problem consists in finding solutions of the following differential equation of the first order of the Riccati type:

$$\frac{dy}{d\mu} = \nu y^2 + \frac{1}{\mu}\left[\nu - 1 + \frac{\nu(\gamma-1)}{2}\frac{\mu^{\nu(\gamma-1)}}{\varkappa + \mu^{\nu(\gamma-1)}}\right]y + \frac{\nu\varkappa(1-\gamma^2)}{4\mu^2[\varkappa + \mu^{\nu(\gamma-1)}]} \quad \left(\varkappa = \frac{A}{B}\right) \qquad (1)$$

The notation is as in reference [1]. In articles [1,3] special cases of equation (1) were examined when $\kappa = 0$, $B = 0$, $\gamma = 1$.

In the present note, a general solution of the problem is given for arbitrary κ and γ. Using the particular solution $y = -(\gamma + 1)/2\mu$, we can obtain the general solutions of equation (1) in the form

$$y = -\frac{\gamma+1}{2}\frac{1}{\mu} + \frac{[(\varkappa + \mu^{\nu(\gamma-1)})]^{1/2}}{\mu^{\nu\gamma+1}[C_1 - J_1(\mu)]}, \qquad J_1(\mu) = \nu\int\frac{[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2}}{\mu^{\nu\gamma+1}}\,d\mu \qquad (2)$$

where C_1 is a constant. For $r_2(\mu)$ we get

$$r_2(\mu) = C_2\mu^{-1/2(\gamma+3)}(C_1 - J_1)^{-1/\nu}$$

where C_2 is a constant of integration. Following the method presented in papers [1,3] it is easy to obtain all the relationships of interest to us

$$q(\mu) = \frac{2[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2}}{2[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2} - (\gamma+1)\mu^{\nu\gamma}(C_1 - J_1)}, \qquad p_2(\mu) = p_1\left\{1 - \frac{\gamma\mu^{\nu\gamma}(C_1 - J_1)}{[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2}}\right\}$$

$$\rho_1(\mu) = \frac{2\gamma p_1}{BC_2^2}\frac{\mu^{2\nu\gamma+\gamma-1}(C_1 - J_1)^{2(\nu+1)/\nu}}{[\varkappa + \mu^{\nu(\gamma-1)}]^{3/2}\{2[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2} - (\gamma+1)\mu^{\nu\gamma}(C_1 - J_1)\}}$$

$$\rho_2(\mu) = \frac{2\gamma p_1}{BC_2^2}\frac{\mu^{2\nu\gamma+\gamma-1}(C_1 - J_1)^{2(\nu+1)/\nu}}{[\varkappa + \mu^{\nu(\gamma-1)}]^{3/2}\{2[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2} - (\gamma-1)\mu^{\nu\gamma}(C_1 - J_1)\}}$$

$$v_2(\mu) = \mp B^{1/2}C_2\mu^{1/2(1-\gamma)}[\varkappa + \mu^{\nu(\gamma-1)}]^{1/2}(C_1 - J_1)^{-1/\nu}$$

The arbitrary function $P(x)$ will then be as follows:

$$P(x) = \frac{2(s+2)}{B\nu(\gamma-1)}\left\{\frac{p_1}{\mu(x)^{\nu\gamma}}\left[1 - \frac{\gamma\mu^{\nu\gamma}(x)[C_1 - J_1(x)]}{[\varkappa + \mu(x)^{\nu(\gamma-1)}]^{1/2}}\right] - C\right\}$$

where $\mu(x)$ is found from the relation

$$x^{\nu/(2+s)}\mu^{1/2\nu(\gamma-1)}[C_1 - J_1(\mu)] - C_2^{\nu} = 0$$

The integral $J_1(\mu)$ can be expressed in terms of elementary functions in only two cases (in addition to the one examined in [1]

$$\gamma = \frac{n_0+2}{n_0+1} \quad \text{or} \quad \gamma = \frac{2n_0+3}{2n_0+1} \qquad (n_0 = 0, 1, 2, \ldots)$$

The solution constructed here can also be obtained by another method [4].

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